Are superconductors really superconducting?

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The most striking difference between the behaviour of the copper oxide high-temperature superconductors and previous low-temperature type II superconductors is the much more gradual decrease in electrical resistance with temperature in the latter, in the presence of a magnetic field. This raises the question of whether a type II superconductor has strictly zero resistivity, when cooled in a magnetic field. Theoretical and experimental evidence now suggests that as the temperature is lowered, there is a sharp phase transition to a truly superconducting, impurity-dominated phase containing a disordered, frozen arrangement of magnetic flux vortices.

In 1911, Kammerlingh-Onnes found that the electrical resistivity of mercury abruptly vanished below a temperature of 4.2 K. This historic observation of superconductivity led to the discovery of a host of related phenomena, including the expulsion of magnetic fields (the Meissner effect) and magnetic flux quantization. Attempts to understand these phenomena culminated in the phenomenological theory of Ginzburg and Landau. The microscopic theory of Bardeen, Cooper and Schreiffer (BCS) provided a justification of Ginzburg-Landau theory and explained many other properties of superconductors[1]. Although most of the macroscopic properties of superconductors were understood[1] by 1970, there remained theoretical problems with understanding the apparent vanishing of the resistivity in a wide class of superconductors: the 'type II' materials. Ironically, the original and defining property of superconductors, zero resistivity, was in some ways the least well understood.

The discovery of the high-temperature (high-T_c) copper oxide superconductors in 1986 led to a resurgence of interest in superconductivity and to the development of a new class of type II materials in which the superconductivity resides on layers of CuO_2. Although interest has focused on the microscopic 'mechanism' of superconductivity in these materials and their peculiar normal-state properties, many of their macroscopic properties are far more strikingly different from previous superconductors. In Fig. 1, the resistivity of $Bi_2Sr_2CaCu_2O_{8+\delta}$ (known as BSCCO) is shown on a logarithmic scale for various magnetic fields as a function of temperature. In zero magnetic field, the resistivity seems to vanish abruptly at a transition temperature of $T_c \approx 90$ K, but in moderate magnetic fields of a few tesla, the resistivity is measurable down to temperatures below 20 K in spite of the expectation that it 'should' have vanished abruptly at a temperature $T_{c2}(H)$ near 80 K. Indeed, in fields greater than 5 T, BSCCO does not become a better conductor than good copper until below 30 K. These data raise fundamental questions. Is BSCCO really superconducting (with truly zero resistivity) at low temperatures in a magnetic field? More generally, do type II superconductors undergo a true phase transition to a state with strictly zero resistivity when cooled in a magnetic field, or does the resistivity just become too small to measure? Here we review recent attempts to answer these and related questions. Although these issues can be addressed entirely within the context of conventional Ginzburg-Landau theory, albeit with unconventional values of parameters, the answers require an understanding of the interplay between strong thermal fluctuations, randomly placed impurities and other crystal imperfections, and transport in disordered media. Each of these areas has seen extensive development of theory and experiment in other contexts during the past two decades. Our current understanding—at least of the right questions if not all the answers about superconductivity—would not have been possible without the advances in these apparently unrelated fields during the period (roughly 1970–85) when research on superconductivity was relatively dormant.

Flux lines and vortices

When a superconductor is cooled below its transition temperature, the electrons pair together to form 'Cooper pairs'. Although the BCS microscopic theory explains why this pairing occurs, the older, simpler and more phenomenological Ginzburg–Landau theory is adequate and indeed necessary to understand many of the most striking features of the superconducting phase[1]. It focuses exclusively on the coherent quantum-mechanical wavefunction, ψ, of the Cooper pairs, its variations in space and time, and its interactions with electric and magnetic fields. This wavefunction is a complex scalar and can thus be characterized by its amplitude and phase: $\psi = |\psi| e^{i\phi}$.

In general both the amplitude and the phase of the wave function will vary in space and time, but it is primarily the spatial correlations in the phase, ϕ, that underlie the striking features of superconductivity. In the absence of a magnetic field, as a material is cooled into the superconducting state, the phase

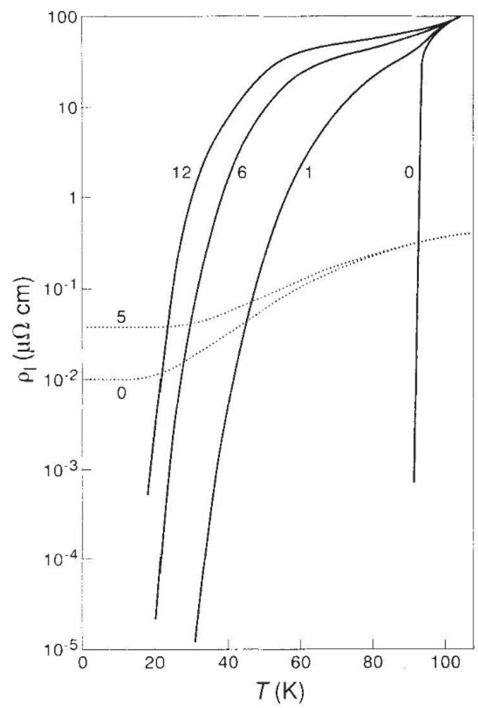

FIG. 1 The temperature (T) dependence of the linear response resistivity, $\rho_l \equiv \lim_{J\to 0}\{E/J\}$, where E is the electric field and J is the current density in the material. The solid curves are for $Bi_2Sr_2CaCu_8O_{8+\delta}$ (BSCCO) and, for comparison, the dotted curves are for high-conductivity copper. Each curve is for a different magnetic field, indicated in tesla. Data are from refs 33–35. For low-temperature superconductors the drop in the resistivity remains fairly abrupt even in a magnetic field (as it is for BSCCO in zero field only).

of the wavefunction becomes macroscopically correlated, taking on the same time-averaged value at every point in the material. Such macroscopic correlations are referred to as 'long-range order', a concept which plays a central role in modern condensed-matter physics.

Another classic example of long-range order occurs in magnetic materials. These come in a myriad of varieties, ranging from ferromagnets and antiferromagnets to the exotic frustrated materials called spin glasses[2]. There are fruitful analogies between the types of magnetic order and the ordered states of a superconductor when it is placed in a magnetic field, as indicated in Fig. 2. The spins of the electrons in a solid can be visualized as tiny microscopic magnets. In a ferromagnet, the interactions between spins cause them to align in the same direction throughout the material. This cooperative phenomenon produces a permanent magnet, with each spin on average pointing towards, say, the north pole of the magnet. Such magnetic long-range order is closely analogous to the long-range order in a superconductor, with the direction of ψ in the complex plane (the phase ϕ) playing the role of the spin's orientation.

One of the classic signatures of a superconductor, and one that is often used in testing new materials for possible superconductivity, is the expulsion of a small external magnetic field—the Meissner effect (Fig. 2). Although superconductors expel small magnetic fields, a sufficiently intense field will penetrate the material, but not without unusual consequences. In type I superconductors, the magnetic field can only penetrate at the cost of destroying the superconductivity. In type II superconductors, there is a compromise: small fields are expelled, but a magnetic field in excess of the lower critical field, H_{c1}, penetrates nonuniformly, forming the 'mixed state'. The magnetic field forms flux lines, each line carrying precisely one unit of magnetic flux, the flux quantum $\phi_0 \equiv hc/2e$ which is set by the total charge $2e$ of the Cooper pairs. The diameter of each flux line is set by a characteristic length, called the magnetic penetration length, and denoted λ. More generally, λ is the length scale over which a magnetic field can vary in a superconductor. Near the middle of each flux line the amplitude, $|\psi|$, of the Cooper-pair wavefunction is suppressed to zero. The region where the amplitude is strongly suppressed is called the vortex core and has a radius of size the coherence length ξ, the second fundamental length characterizing a superconductor. In the high-T_c copper oxides, ξ, which is in the range 10–20 Å at low temperatures, is much smaller than the penetration length, λ, which exceeds 1,000 Å (ref. 3). In low-T_c type II superconductors, ξ is usually much larger. The ratio $\kappa \equiv \lambda/\xi$ is an important dimensionless parameter. In Ginzburg–Landau theory, the value $\kappa = 1/\sqrt{2}$ marks the boundary between type I and type II materials. The high-T_c cuprates, with very large κ, are extreme type II superconductors. Here we focus only on type II superconductors, which enter the mixed state when a magnetic field penetrates the material.

On fully encircling a vortex outside its core, the phase ϕ of the wavefunction changes (winds) by 2π. This phase change reflects electric currents (supercurrents) that flow around the vortex, screening the magnetic field and confining it to within a distance λ of the vortex core. In contrast to vortices in classical fluids, the vorticity in a superconducting vortex is quantized; the phase change around a vortex must be an integer multiple of 2π because the wavefunction ψ is single-valued. Because of this quantization, a superconducting vortex is topologically stable, and a vortex line cannot terminate in the interior of a superconductor.

Abrikosov[4] elucidated the nature of the vortex or flux lines and predicted that when a type II superconductor is placed in a magnetic field in excess of H_{c1}, the vortices that penetrate the material should form a regular lattice. This prediction was confirmed a decade later[5] in magnetic decoration experiments that showed a triangular array of flux lines, an Abrikosov vortex lattice (Fig. 2). If one neglects thermal fluctuations[6], the Abrikosov vortex lattice shows two kinds of long-range order, although as we shall see, neither is robust enough to survive the ever-present imperfections and fluctuations in real materials. The most apparent are the long-range translational (and orientational) correlations present in the vortex array itself, directly analogous to crystalline correlations in a solid. A more subtle long-range order is also present: the phase, ϕ, of the wavefunction is ordered in a nontrivial spatial pattern, reflecting the underlying vortex lattice. This long-range phase coherence is analogous to antiferromagnetic order in magnetic materials, in which the electron spins show order with a periodic spatial structure (see Fig. 2).

If the magnetic field in the vortex lattice is increased, the vortices become more closely spaced and their cores start to overlap. At the upper critical field the vortex lattice and the pairing of electrons disappear, and the material becomes 'normal'. Neglecting thermal fluctuations, the upper critical field is $H_{c2} = \phi_0/(2\pi\xi^2)$, so small coherence lengths give rise to large upper critical fields. For the high-T_c cuprate superconductors at low temperatures, the small coherence lengths should yield upper critical fields exceeding 100 T, greater than the fields available in today's magnets. A full phase diagram of a clean type II superconductor, obtained from a 'mean-field' treatment of Ginzburg–Landau theory, is shown in Fig. 3*a* for a particular value of κ. Similar phase diagrams are quantitatively good for most clean low-T_c type II superconductors. But thermal fluctuations and sample imperfections, which were omitted in deriving Fig. 3*a*, can greatly change this phase diagram[7].

Thermal fluctuations

In low-T_c superconductors, thermal fluctuations generally have a minor role, but thermal fluctuations are much more important

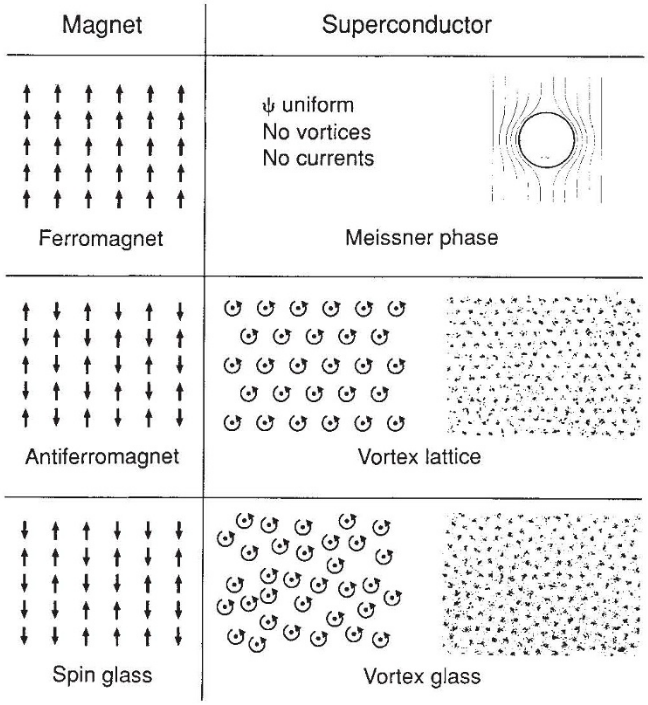

FIG. 2 Patterns of spin order in magnetically ordered systems (left, where arrows indicate the orientation of the local spins) and the analogous ordered phases of a superconductor (right). In the Meissner phase of the superconductor (top) the magnetic field and the vortices are expelled from the material, as indicated by the magnetic field lines around an ideal spherical superconducting ball. For the vortex lattice and glass, the positions of the vortex cores are indicated by dots and the circulating currents by arrows in the schematic drawings; on the right are images of vortex patterns on BSCCO crystals obtained by decorating the surface with fine magnetic particles that preferentially stick near the magnetic flux lines on each vortex. The decoration images are from the work reported in ref. 36.

in the high-T_c copper oxide superconductors. This is because the temperatures are higher and the energies to create and move vortices are lower. The relevant energy scale is that required to create a piece of vortex line whose length is one coherence length[7]. This energy scale varies as ξ/λ^2, so it is much lower in the high-T_c materials with their small coherence lengths and large penetration lengths. The coherence length, ξ, is roughly the size of the bound Cooper pairs of electrons, which are much smaller in the high-T_c materials, and the large penetration lengths reflect weak supercurrents because of the relatively low density of mobile electron pairs.

The most striking effects of enhanced thermal fluctuations in the high-T_c superconductors are found in an applied magnetic field. The phase diagram for a type II superconductor with strong thermal fluctuations is shown in Fig. 3*b*. Notice the vortex-fluid regime between the mean-field upper critical field, $H_{c2}(T)$, and the vortex-glass phase. On cooling in a field, the electrons start to pair and vortices form in the pair wavefunction near H_{c2}, but the vortices do not freeze until substantially lower temperatures. The existence of a substantial vortex-fluid regime had been noted for thin superconducting films[8,9], but it was not observed in bulk superconductors until the copper oxide superconductors had been studied[10,11]. In the vortex fluid, vortices are mobile and have only short-range correlations in their positions, much like atoms in a conventional fluid. The vortex-fluid regime extends to particularly low temperatures in extremely anisotropic layered materials (like BSCCO) when the magnetic field is directed perpendicular to the layers[12,13]. In this case, the vortex lines actually consist of strings of point or 'pancake' vortices in each superconducting layer, with only rather weak correlations between vortices in different layers. The presence of a large vortex-fluid regime explains quite naturally the resistivity data for BSCCO shown in Fig. 1.

The vortex fluid is the natural analogue of the disordered, paramagnetic phase that occurs in magnetic materials heated above the Curie (or Neel) temperature. As in a paramagnet, the vortex fluid does not have any long-range order: vortex motion 'scrambles' the phase ϕ of the pair wavefunction, disrupting any possible long-range phase coherence and destroying superconductivity. It should be emphasized that the vortex fluid is not really a distinct phase: its properties evolve smoothly and continuously as the field or temperature is increased through H_{c2} to the normal state.

Resistivity

Which of the phases in the phase diagrams (Fig. 3) are actually superconducting, with zero resistance? Although zero resistance is in many ways the defining characteristic of a superconductor, it is in some sense the hardest to explain. A crucial part is played by the motion of vortex lines, which causes dissipation and thus resistivity. In a superconductor the local electrical current flows in the direction in which the phase ϕ increases and the current density **J** is proportional to the rate of increase of ϕ along that direction: $\mathbf{J} \propto \nabla\phi$. This current exerts a Magnus force on a vortex line, pushing it across the current (a closely related effect gives an aerofoil upward lift when a wind blows across it). The total change of phase ϕ on passing through a superconducting material on one side of a vortex line differs by 2π from that found on passing on the other side. When vortex lines move across the material in response to the Magnus force they move in the direction that reduces the total change of phase and thus reduces (dissipates) the current. To maintain a steady current, a voltage difference (and thus an electric field E) must be maintained across the material. The voltage difference acts to increase the phase difference across the material, balancing the decrease due to the motion of the vortex lines, and thus maintaining a steady current. In the Meissner phase ($H < H_{c1}$) the magnetic field is expelled from the material and there are no free vortex lines present to move and cause resistivity. Thus the Meissner phase is indeed superconducting with zero linear resistivity $\rho_l \equiv \lim_{J\to 0}\{E/J\} = 0$. Note that ρ_l is the ohmic resistivity which measures the linear response to an applied current or field. As we shall see later, even in the Meissner phase there is some nonlinear resistivity: a nonzero electric field is needed to maintain any nonzero current density in the interior of the material, so the nonlinear resistivity $\rho \equiv E/J$ only vanishes in the limit of zero current density.

In a superconductor in the mixed state ($H > H_{c1}$), there are vortex lines present, induced by the penetrating magnetic field. Motion of these lines leads to resistivity. In the vortex-fluid regime the vortex lines are mobile because of thermal fluctuations even without a current, and can thus move in response to a current, leading to a nonzero resistivity. But what about in the Abrikosov vortex-lattice phase? Perhaps surprisingly, in a perfectly clean and ideal material the entire vortex lattice would be free to move in response to a current, and again cause nonzero resistivity. In an ideal superconductor, therefore, the Abrikosov vortex-lattice phase is not really superconducting. Nevertheless, real materials always have chemical and structural imperfections ('dirt') and these can impede the motion of the vortex-lattice[14]. In the presence of a penetrating magnetic field, if we wish to answer the question posed in our title, we must consider the role played by dirt.

Dirt and the vortex-glass phase

More than 20 years ago, Larkin[15] argued that material imperfections destroy the crystalline long-range order of the Abrikosov vortex lattice. There are two competing effects: the interactions between the vortex lines favour a lattice structure, but the

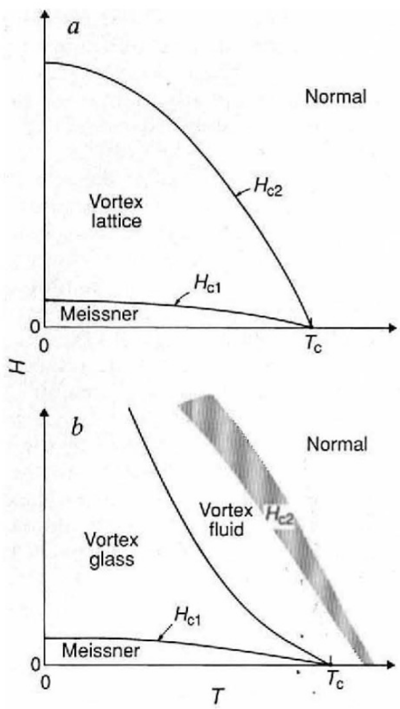

FIG. 3 *a*, Phase diagram of a type II superconductor, ignoring effects of thermal fluctuations and material imperfections, as a function of temperature, *T*, and applied magnetic field, *H*. This phase diagram is qualitatively correct for most low-T_c type II superconductors. *b*, Schematic phase diagram for a type II superconductor with strong thermal fluctuations, such as the high-T_c cuprates. In the absence of material imperfections the low-temperature ordered phase in a magnetic field above H_{c1} would have been a vortex lattice but the ever-present imperfections destabilize the vortex lattice, replacing it with the vortex glass phase, as shown. The transition at H_{c2} in *a* has here been broadened by fluctuations and has become only a gradual crossover from a normal metal to a vortex fluid, indicated by the shaded region.

randomly located material defects tend to pin the lines at random positions, disrupting the lattice structure. Larkin argued that beyond a characteristic length scale, l_L, the random vortex pinning dominates and destroys the vortex-lattice phase. Although the size, l_L, of the microcrystalline regions can be very long for weak pinning, the long-range lattice (positional) correlations are destroyed for distances in excess of l_L.

The conventional theory of vortex motion, based on ideas originated in the early 1960s by Anderson and Kim[14,16,17], maintains that finite 'bundles' of vortex line segments of length and diameter of order l_L can move by thermal activation across the 'landscape' of free energy barriers caused by the pinning. In this approach the interactions between the vortex bundles are ignored. Because the bundles are finite, the barriers have a finite maximum height, U. The independent motion of vortex bundles forced by an applied current would destroy the phase coherence of the pair wavefunction and lead to a finite resistivity with a thermally activated (Arrhenius) form: $\rho_l \sim \exp(-U/k_B T)$. Thus the macroscopic behaviour of vortices in the mixed state was predicted to be that of a very sluggish vortex fluid (like cold molasses flowing through a sponge). According to this theory, superconductors in a magnetic field are not really superconducting, except at zero temperature. For practical purposes, however, this conclusion is almost irrelevant for conventional superconductors: $U/k_B T$ is so large that ρ_l is immeasurably small except extremely close to the H_{c2} line.

For the high-T_c copper oxide superconductors, on the other hand, $U/k_B T$ is not large and one must reconsider the issue, particularly in light of the new experimental measurements (for example those in Fig. 1). We have recently explored a different scenario[7,18] from that envisaged by Anderson and Kim. We argue that on cooling, the fluid of vortex lines interacting with the random pinning sites undergoes a sharp thermodynamic phase transition into a truly superconducting phase with no mobile vortices and thus strictly zero resistivity. In this superconducting phase the vortices are frozen into a sample-specific compromise configuration, determined in detail by the competition between the interactions of the vortices with each other and with the microscopic impurities in the material. Because the vortices are immobile, the phase of the pair wavefunction has true long-range order, although in a seemingly random pattern which reflects in detail the specific positions of the frozen vortices. The order present in this superconducting phase is directly analogous to the magnetic order that occurs in some random magnetic alloys, called spin glasses (see Fig. 2). In spin glasses[2], the spin axes of the electrons are frozen in time, but rather than being aligned as in a ferromagnet, or in a spatially periodic pattern as in an antiferromagnet, they are oriented in a sample-specific arrangement, determined by the microscopic details of the material. Because of this analogy, the new superconducting phase has been named the 'vortex glass'[19].

In contrast to the Meissner phase, the superconducting vortex-glass phase can only exist in an impure material. Thus we argue that superconductors in a penetrating magnetic field can be truly superconducting only when they are dirty. The random pinning which is present to some extent in any real material, no matter how carefully prepared, destabilizes the vortex-lattice phase (which was not truly superconducting) and converts it into the superconducting vortex-glass phase.

In our view, the fundamental flaw in the theories of thermally activated vortex motion based on Anderson and Kim's work[14,16,17] is that they assume independent motion of finite bundles of vortex-line segments. But as mentioned above, a vortex line cannot end in a superconductor, so the vortex lines are constrained to remain connected as segments of them are moved. We argue that at low temperatures this constraint and the interactions between the vortex lines (which extend out to distances of the order of the penetration length, λ) cannot be ignored, even on length scales well beyond the microcrystallite size l_L. These interactions cause phase correlations which at sufficiently low temperature can propagate to arbitrarily long distances, producing the long-range order of the vortex-glass phase.

We will now consider some of the key predictions of the theory of the vortex-glass phase and their experimental confirmation. The evidence for the vortex-glass phase comes from transport experiments on the copper oxide superconductors conducted over the past few years[20-24].

Vortex-glass phase transition

A fruitful way to distinguish a truly superconducting phase, with strictly zero resistivity, from a vortex fluid with immeasurably small but nonzero resistivity is to search for a phase transition as the temperature is lowered. If the superconducting vortex-glass phase exists, there must be a sharp thermodynamic phase transition which separates it from the (nonsuperconducting) vortex fluid. This phase transition may be of first order, in which case the resistivity will jump discontinuously to zero and other properties will be discontinuous at the phase transition, or it may be continuous (second order). In the latter case, which seems to obtain for samples of $YBa_2Cu_3O_7$ (YBCO) with strong enough random pinning, one expects universal critical scaling behaviour for temperatures near the vortex-glass phase transition temperature, T_{vg}.

Critical scaling behaviour occurs in all continuous phase transitions[25] and can be attributed to the existence of a characteristic length which diverges at the transition. Above the vortex-glass transition, the relevant length, ξ_{vg}, is the length scale over which the phase of the pair wavefunction is correlated (albeit in a random pattern). As at most continuous phase transitions, one expects a power-law divergence of the correlation length as the transition temperature is approached: with $\xi_{vg} \sim |T - T_{vg}|^{-\nu}$, characterized by a critical exponent ν. Critical slowing of the dynamics is also expected near a continuous phase transition, with the relaxation time, τ, diverging as a power of ξ_{vg}, $\tau \sim \xi_{vg}^z \sim |T - T_{vg}|^{-z\nu}$, where z is the dynamical critical

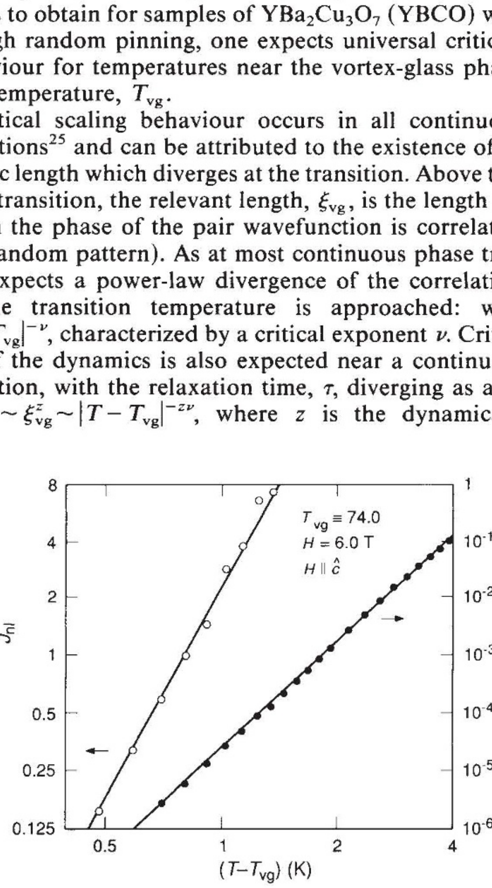

FIG. 4 Scaling in the vortex-glass critical regime for a crystal of YBCO in a 6-T magnetic field applied parallel to the material's $\hat{c}$-axis (perpendicular to the CuO_2 planes). Data are from ref. 21. The solid circles are the linear resistivity, ρ_l, divided by the resistivity in the normal state, ρ_n, plotted on logarithmic scales against the difference of the temperature, T, from that of the vortex-glass phase transition, $T_{vg} = 74$ K. The slope of the solid line through the solid circles yields the critical exponent $s = \nu(z-1) \approx 6.5$. The open circles are the current density, J_{nl} (in arbitrary units), for the onset of nonlinearity in the resistivity. Here the slope yields $2\nu \approx 4$.

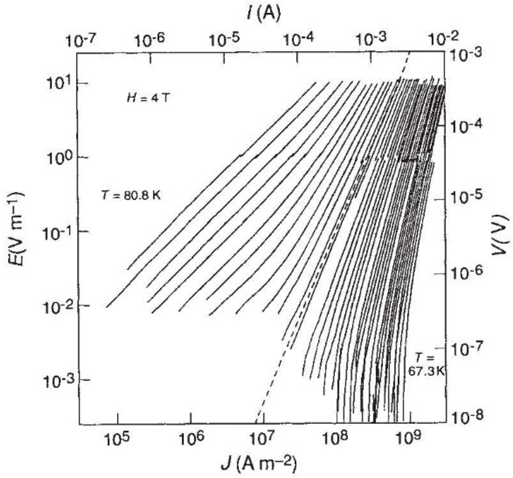

FIG. 5 Current–voltage ($I-V$) curves at fixed temperatures for a 0.4-μm-thick YBCO epitaxial film in a 4-T magnetic field applied parallel to the material's $\hat{c}$-axis. Adjacent curves differ by temperature intervals of ~0.3 K, with the highest and lowest temperatures as indicated. The curves are on logarithmic scales, and the current densities J and electric fields E are also indicated. The dashed line indicates the vortex-glass transition temperature at $T_{vg} \approx$ 77 K at which $E \sim J^{(z+1)/2}$ with $z \approx 4.8 \pm 0.2$. For temperatures above T_{vg} at low current densities, $E \sim J$, indicating the ohmic behaviour characteristic of a normal metal or vortex fluid. Below T_{vg}, the slope of the curves becomes steeper and steeper at low voltages suggesting exponentially small dissipation as predicted for the vortex-glass phase. Data are from ref. 20. The curves were measured in separate sweeps up to $E \approx 1$ V m^{-1} and to $E \approx 10$ V m^{-1}. The breaks between the two sets of curves at $E \approx 1$ V m^{-1} are due to resistive heating in the larger E sweeps.

exponent. This time is, roughly speaking, the time it takes a fluctuation of size ξ_{vg} to relax. The renormalization group theory of critical phenomena near continuous phase transitions[25] predicts that the critical exponents are universal numbers which characterize the type of transition but are insensitive to microscopic details, such as the material. In addition, one expects there to be universal scaling laws which can relate measurements at different temperatures near the transition, provided each physical quantity is scaled by the appropriate power of $(T - T_{vg})$. In particular we will focus on the dissipation as a function of applied current: the current–voltage curves. The linear resistivity is predicted to vanish as $\rho_l \sim (T - T_{vg})^s$ as the transition is approached from above, with $s = \nu(z-1)$, an example of a scaling law[7]. Such a power-law form with exponent $s \approx 6.5$ for the resistivity has been observed for a sample of YBCO over a temperature range where the resistivity drops by more than four orders of magnitude[21] (Fig. 4).

At temperatures just above T_{vg}, the vortex-fluid state has a very long correlation length ξ_{vg}. But these correlations can be disrupted by the vortex-line motion caused by even a small applied current density J, so the nonlinear resistivity $\rho(J) \equiv E/J$ is very sensitive to the magnitude of J at temperatures near T_{vg}. It is useful to define a characteristic current-density scale J_{nl} as that current at which the vortex-fluid state is significantly disrupted. To be specific, for $T > T_{vg}$ we choose J_{nl} as the current at which the nonlinear resistivity doubles: $\rho(J_{nl}) = 2\rho_l$. We predict[7] that J_{nl} should vanish as the inverse square of the correlation length, $J_{nl} \sim \xi_{vg}^{-2} \sim (T - T_{vg})^{2\nu}$; the measurements of Gammel *et al.*[21] on YBCO yield $2\nu \approx 4$ (Fig. 4). This rapid drop of J_{nl} as T approaches T_{vg} from above is the experimental result most strikingly inconsistent with the 'conventional' theories[14,16,17] of thermally activated vortex motion. These theories assume that the correlation length is roughly Larkin's crossover length l_L, so they do not predict any strong temperature dependence of J_{nl} in this regime. The 'conventional' approach therefore fails at temperatures and current densities low enough that the correlation length, ξ_{vg}, is greater than the size of the microcrystalline regions, l_L.

In addition to predicting these power-law behaviours, scaling theory predicts that the functional dependence of the electric field on the current density will be the same for any temperature near but above T_{vg}, provided that the dissipation is normalized by the linear resistivity and the current density is normalized by J_{nl}

$$\frac{\rho(J)}{\rho_l} \equiv \frac{E}{J\rho_l} \approx \mathscr{R}_+\left(\frac{J}{J_{nl}}\right) \qquad (1)$$

where $\mathscr{R}_+(j)$ is a universal scaling function with $\mathscr{R}_+(j \to 0) = 1$ and, from the above definition of J_{nl}, $\mathscr{R}_+(1) = 2$. As we decrease T to T_{vg} at fixed J, both $\rho(J)/\rho_l$ and J/J_{nl} diverge. For E and $\rho(J)$ at T_{vg} to be well-behaved, the scaling function in equation (1) must behave as $\mathscr{R}_+(j) \sim j^{(z-1)/2}$ for large j. Then at T_{vg} one finds $\rho(J) \sim J^{(z-1)/2}$ or $E \sim J^{(z+1)/2}$, a power-law relationship between voltage and current. Current–voltage ($I-V$) curves measured by Koch *et al.*[20] on YBCO show the general features of the continuous phase transition (Fig. 5). The power-law behaviour at T_{vg} is indicated by the dashed line. For $T > T_{vg}$ and $J \ll J_{nl}$, ohmic behaviour with $E \approx \rho_l J$ is observed, whereas for larger currents the nonlinearity becomes appreciable. Both ρ_l and J_{nl} decrease continuously as T approaches T_{vg} from above. The values of the critical exponents extracted from the two experiments[20,21] described above are in good agreement, although the samples and current densities studied were different. Moreover, recent computer simulations[26,27] of a model superconductor reveal evidence for a vortex-glass phase transition with similar critical exponents.

Non-ohmic resistance

Just below the vortex-glass transition at T_{vg} there is a scaling behaviour[7] analogous to equation (1), with a characteristic current scale J_{nl}. But now the behaviour for $J \ll J_{nl}$ is not ohmic; rather, as the current is decreased the voltage vanishes faster than any power of the current, as indicated by the curvature to very high slopes at the bottom right of Fig. 5. This dissipation regime is that due to vortex motion in the ordered vortex-glass phase. To understand this, let us first consider the analogous dissipation processes in the much simpler Meissner phase.

In the Meissner phase ($H < H_{c1}$) there are no free vortices present, but at nonzero temperatures thermal fluctuations spontaneously occur, typically making local excitations with free energy of the order of the thermal energy, k_BT. One type of excitation that occurs and can lead to dissipation of a current

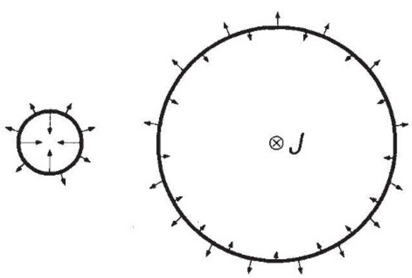

FIG. 6 The bold circles represent closed vortex loop excitations in the Meissner phase. The inward-pointing arrows denote the inward force on the loop due to the line tension of the vortex; this force is proportional to the curvature of the loop. In the presence of a current density passing through the loop (into the page) there is also an outward Magnus force on the loop due to its interaction with this current (the opposite current would produce an inward force). The loop on the left is smaller than the critical radius so the inward force is larger and the loop tends to contract. The loop on the right is larger than the critical radius so the outward force dominates, and it tends to grow still larger, contributing to the dissipation of the current.

is closed vortex loops of finite circumference[28] (Fig. 6). The free energy, $f_V(R)$, of a circular vortex loop of radius R is roughly proportional to its circumference, $f_V(R) \approx 2\pi R\varepsilon$, where ε is the free energy per unit length or, equivalently, the line tension of a vortex. The Boltzmann probability of this loop appearing as a spontaneous thermal fluctuation at temperature T is thus $\exp[-f_V(R)/k_BT]$. This is the probability that random thermal fluctuations coincidentally produce outward forces on the loop of sufficient average strength and duration for it to grow to this size. In the absence of any applied current such a vortex loop will almost certainly rapidly shrink in response to the ever-present inward force of ε/R per unit length arising from its line tension and curvature. Let us assume, however, that there is a uniform current density J passing through the loop. The outward Magnus force from this current can counterbalance the inward force from the vortex line tension (Fig. 6) and cause loops of radius greater than a critical size $R_c(J) \approx \varepsilon/J$ to tend to grow indefinitely; the resulting vortex motion produces dissipation[28]. (The process of nucleation of vortex loops bigger than the critical radius $R_c(J)$, is analogous to bulk nucleation of droplets of liquid in a supersaturated vapour, the applied current playing the role of the degree of supersaturation and R_c the role of the critical droplet radius.) The dissipation rate in the Meissner phase is proportional to the probability of nucleating sufficiently large loops, $\exp[-f_V(R_c(J))/2k_BT]$, yielding, for small current density J

$$E/J = \rho(J) \approx \exp[-(J_T/J)^{\mu}] \quad (2)$$

with $\mu = 1$ and $J_T \approx \varepsilon^2/k_BT$ setting the current scale of this thermally activated dissipation. Thus there is always dissipation, even in the Meissner phase, but at small current densities it arises from rare large thermal fluctuations and vanishes exponentially for $J \to 0$.

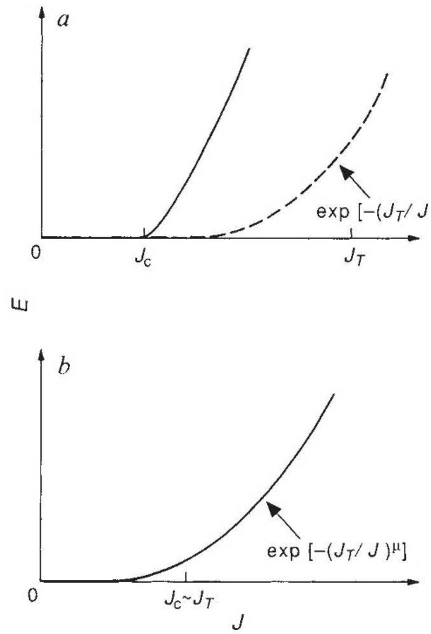

FIG. 7 Electric field E against current density J in a truly superconducting phase (a Meissner or vortex-glass phase). *a*, With weak thermal fluctuations, $J_T \gg J_c$ and the electric field for $J < J_c$ is small. The dashed line is the extrapolation of the activated behaviour for $J < J_c$ to $J > J_c$. Near J_c, however, the activation barriers for vortex motion vanish and there is a rapid increase in the dissipation, leading to an apparently sharp critical current. This behaviour is found in low-T_c superconductors. *b*, With strong thermal fluctuations, J_c and J_T are comparable and the onset of dissipation is more gradual. This behaviour is found in the high-T_c superconductors (except at temperatures far below T_c where the behaviour is more like that in *a*).

How does this fit in with the conventional concept of a 'critical current' in superconductors? For small current densities, the free energy barrier that must be crossed to produce a vortex loop varies as $J^{-\mu}$. But this barrier becomes of order k_BT at the critical current J_c and the thermally activated dissipation law of equation (2) thus only applies for $J < J_c$; for $J > J_c$ vortices are readily produced, yielding strong dissipation. In low-T_c superconductors, $J_c \ll J_T$ except very near T_c, so the nucleation of loops is very rare for $J < J_c$. Thus one finds a fairly abrupt increase in the dissipation at J_c (Fig. 7*a*). Very near T_c in conventional superconductors, and over large portions of the phase diagram of the high-T_c copper oxides, on the other hand, J_c and J_T are comparable so the onset of dissipation is more gradual, with the critical current ill defined (Fig. 7*b*).

In the vortex-glass phase, in which many vortices penetrate the sample, qualitatively similar dissipative behaviour occurs, although the details are much more subtle and not yet fully understood. It is not clear what the precise character of the dominant vortex-line rearrangements leading to nonlinear dissipation will be in the vortex-glass phase; several proposals have been analysed[7,29-31]. In each of the proposed dissipative processes, however, the free energy barriers that must be crossed to produce the vortex rearrangements grow as a power of the size of the rearranged region, just as in the Meissner phase. If a current is applied through the superconductor, it will induce the thermally activated production of such rearrangements larger than a critical size $R_c(J)$. This will cause phase slip and hence dissipation as in the Meissner phase leading again to a low-current resistance of the form $E/J \approx \exp[-(J_T/J)^{\mu}]$, but now with an exponent $0 < \mu \leq 1$ determined by the nature of the dissipative processes. Such thermally activated vortex motion is often called vortex (or flux) creep. As in the Meissner phase, a sufficiently strong current can overcome the free energy barriers to vortex motion, causing vortex (or flux) flow and rapid dissipation: this will occur above the 'critical' current density J_c. We thus conclude that both the Meissner and vortex-glass phases are linear superconductors with vanishing resistivity only in the linear-response limit of zero current density. For any nonzero current density, the resistivity is nonzero because of thermally activated vortex motion, although it vanishes as an exponential function of the current density, J, for $J \to 0$.

Because of the very small voltages that occur in the activated regime, direct measurement of the behaviour in equation (2) and experimental determination of the exponent μ is difficult. Recent measurements[20,23,24] in the vortex-glass phase, however, yield reasonable fits to equation (2) with values of μ less than 1/2.

Another way to study nonlinear dissipation in the vortex-glass phase is through magnetization relaxation at low temperatures. In a typical experiment a sample is cooled into the vortex-glass phase in a magnetic field, and then the field strength is suddenly changed. This change induces a non-equilibrium screening current, $J(t)$, which decays to zero as vortex lines enter or leave the material and move to their new equilibrium configuration. The slow decay of this current, which can be measured through the associated magnetization, is given by $dJ(t)/dt \propto E(J)$, with $E(J)$ the electric field needed to sustain the current J in steady state. In conventional superconductors (and copper oxide superconductors at very low temperatures) dJ/dt is so small for $J < J_c$ that the current density never drops much below J_c, even in a year. With the strong fluctuations in the copper oxide superconductors, by contrast, the current can decay much further; and the predicted form for $E(J)$ in equation (2) yields a decay as $J(t) \approx J_T[\ln(t/t_0)]^{-1/\mu}$ at very long times, with t_0 a microscopic time of order $10^{-9} - 10^{-12}$ s. This technique gave Sandvold and Rossel[23] a voltage sensitivity almost 10 orders of magnitude better than conventional transport measurements, corresponding to electric fields down to about 10^{-14} V cm^{-1}. Their inferred $E(J)$ is shown in Fig. 8. They find good fits to equation (2) in YBCO films with $\mu \approx 1/3$. This is consistent with the transport

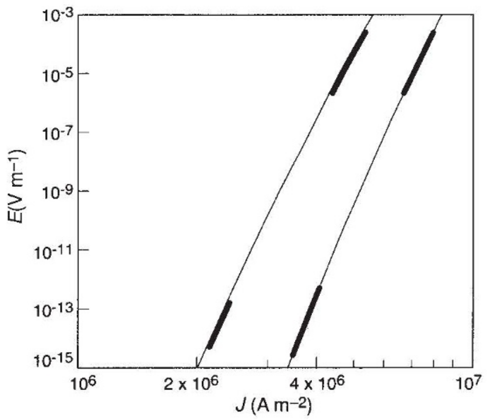

FIG. 8 Electric field E against current density, J on logarithmic scales, as measured for a 0.2-μm-thick epitaxial film of YBCO grown on $SrTiO_3$ (100)[23]. The applied magnetic fields are 0.3 T (left) and 0.1 T (right) and the temperature is 70 K, which is in the vortex-glass phase. The data are the thick curves; at high E the electric field was directly measured, whereas at very low E it was inferred from the slow decay with time of circulating currents in the material. The fits to equation (2) with $\mu = 0.34$ are indicated by the fine lines.

measurements and, together, these experiments provide strong evidence that the resistance vanishes faster than any power of the current in the vortex-glass phase.

Conclusions

We have seen that the combined effects of thermal fluctuations and impurities lead to phase diagrams and transport properties of the high-T_c copper oxide superconductors in a magnetic field that are radically different from those of low-T_c type II superconductors. If a conventional low-T_c type II superconductor is cooled in a magnetic field in excess of H_{c1}, the resistivity abruptly becomes immeasurably small as the $H_{c2}(T)$ line in Fig. 3*a* is crossed. In striking contrast, because of the strong thermal fluctuations in the copper oxide materials, there is no well defined H_{c2} line but only a gradual pairing of the electrons into Cooper pairs leading to the formation of vortex lines (Fig. 3*b*). In the resulting non-superconducting vortex-fluid regime, as the temperature is lowered the vortex motion is increasingly impeded by impurities so the resistivity decreases. This continues until the vortex-glass phase transition, $T_{vg}(H)$, where the linear ohmic resistivity finally vanishes. Near T_{vg} scaling behaviour of current–voltage curves is found, providing the best evidence for a true phase transition. In the vortex-glass phase the dissipation of a small current is dominated by the thermal activation of large collective vortex rearrangements which cause a resistance that vanishes exponentially with current at small currents. At very low temperatures, this thermally activated dissipation is small, and current–voltage curves show a fairly abrupt onset of dissipation at the critical current, with resistance too small to be directly measured at lower currents; this regime is similar to low-T_c superconductors.

Although experiments are qualitatively, and in some respects quantitatively, in accord with the theoretical picture presented here, there remain many open questions and new regimes to be explored. These include the dependence of T_{vg} and the current scales J_T and J_c on the material and its type and distribution of imperfections, one interesting limit being that of very few imperfections (the clean limit). In addition, the nonlinear dissipation in the mixed state has not been carefully examined in either highly anisotropic, quasi-two-dimensional materials such as BSCCO or in isolated, truly two-dimensional superconducting thin films; the latter are expected to be truly superconducting only for zero magnetic field or zero temperature, with no vortex-glass phase at nonzero temperature[32]. Another interesting avenue is the study of artificially structured superconductors, as produced by microfabrication techniques such as molecular-beam epitaxy, where the properties of the structure might be systematically and continuously varied, for example by introducing nonsuperconducting layers of different thicknesses.

Finally, we should directly answer the question posed in our title. Thanks to experiments on the high-T_c copper oxide superconductors and to new theoretical ideas, the answer that was thought to be "no, superconductors in the mixed state are not really superconducting; their resistance is just extremely small" is in fact, "yes, they are really superconducting at temperatures below the phase transition into the vortex-glass phase". □

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